Desynchronization in coupled systems with quasiperiodic driving

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We describe the development of coexisting attractors in coupled quasiperiodically forced maps. The process of loss of complete synchronization in the systems, which individually demonstrates strange nonchaotic behavior, is studied. With this process, the complex structure of the basin of attraction is observed.

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The interaction between coupled chaotic oscillations can produce a number of different bifurcational phenomena. Coupled units following the period-doubling route to chaos exhibit a hierarchy of bifurcations in which different families of attractors emerge [1–3] and the nested structure of the synchronization region is observed [4].

For interacting chaotic oscillators provided that they are identical, complete (full) synchronization can take place [5–7]. That is, when the coupling is appropriate oscillations of the systems coincide completely $\mathbf{x}_1(t)=\mathbf{x}_2(t)$ (i.e., systems oscillate in phase). The transition to nonsynchronous behavior in systems with the symmetric invariant manifolds is associated with a variety of new phenomena including the riddled basin of attraction [8] and on-off intermittency [9]. The loss of complete chaotic synchronization is directly related to bifurcations of saddle periodic orbits embedded in a chaotic attractor [10].

In recent years, evidence for the occurrence in quasiperiodically forced systems (i.e., forced by the signal with two incommensurate frequencies) of a strange nonchaotic attractor (SNA) has accumulated [11–19]. The SNA is characterized by the fractal structure, but the exponential divergence of trajectories does not appear. Recently, in [18] it has been shown that the loss of the transverse stability of an ergodic torus in a certain class of systems with the invariant manifold can lead to strange nonchaotic behavior. This transition is accompanied by blowout bifurcation and on-off intermittency. Ramaswamy [20] has found that quasiperiodically driven systems in a regime of a SNA can be fully synchronized.

In this Brief Report we investigate how a quasiperiodic forcing affects the destruction of the complete synchronization. We demonstrate that the regime of an ''in-phase'' synchronous SNA can be destabilized with the formation of a riddledlike structure of the basin of attraction.

The model of coupled logistic maps under an identical quasiperiodic forcing applied to each system, is given by

$$x_{n+1}=\varepsilon-x_n^2+\gamma(x_n^2-y_n^2)+a_0\cos 2\pi z_n,$$

$$y_{n+1}=\varepsilon-y_n^2+\gamma(y_n^2-x_n^2)+pa_0\cos 2\pi z_n, \qquad (1)$$

$$z_{n+1}=z_n+W, \quad \mathrm{mod}\ 1,$$

where x, y are dynamical variables, z is the phase of the external force with $W=0.5(\sqrt{5}-1)$ irrational, ε represents the nonlinearity parameter, γ governs the coupling strength, a_0 is the forcing amplitude in the first system, while p determines an amplitude mismatch of the interacting systems.

The unforced coupled logistic maps, which individually follow the period-doubling route to chaos, demonstrate the universal scenario of the development of periodic and chaotic coexisting regimes [2]. When an identical ($p=1$) quasiperiodic forcing is applied, a few notable effects are recognized: (i) it transforms the periodic attractors into quasiperiodic ones; (ii) it truncates the period-doubling cascade and provides a transition to the SNA, and (iii) it reduces a number of coexisting attractors and leads to the destruction of multistability as the external amplitude increases. For small forcing amplitude and at fixed parameters $\gamma=0.002$ and $\varepsilon=1.2$, two families of attractors are observed: ''in-phase'' attractors, which are located in the invariant subspace defined by the condition $x=y$, and ''out-of-phase'' attractors with the self-symmetry $x\leftrightarrow y$. At $a_0=0.1$, quasiperiodic attractors of two families coexist in the phase space of the system. With increasing amplitude a_0, the truncated torus-doubling cascade takes place for both families. Variation of control parameters leads to the transition to the SNA.

The appearance of the ''in-phase'' strange nonchaotic attractor SNA_0 occurs at $a_0\approx 0.1185$ when a period-doubled torus collapses with its unstable parent torus [13]. As a collision is approached, the torus becomes wrinkled, ultimately getting a fractal structure at the collision. The ''out-of-phase'' strange nonchaotic attractor SNA_1 with two bands arises at $a_0\approx 0.1182$ via a gradual fractalization of the doubled torus as described in [15–17]. The strangeness of

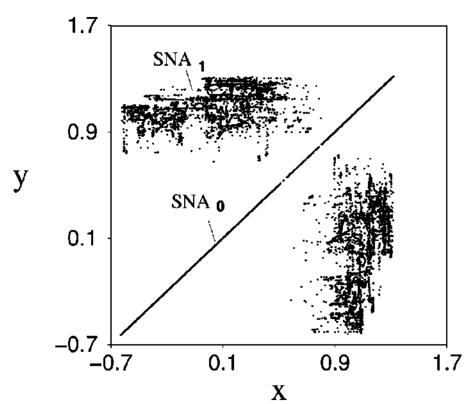
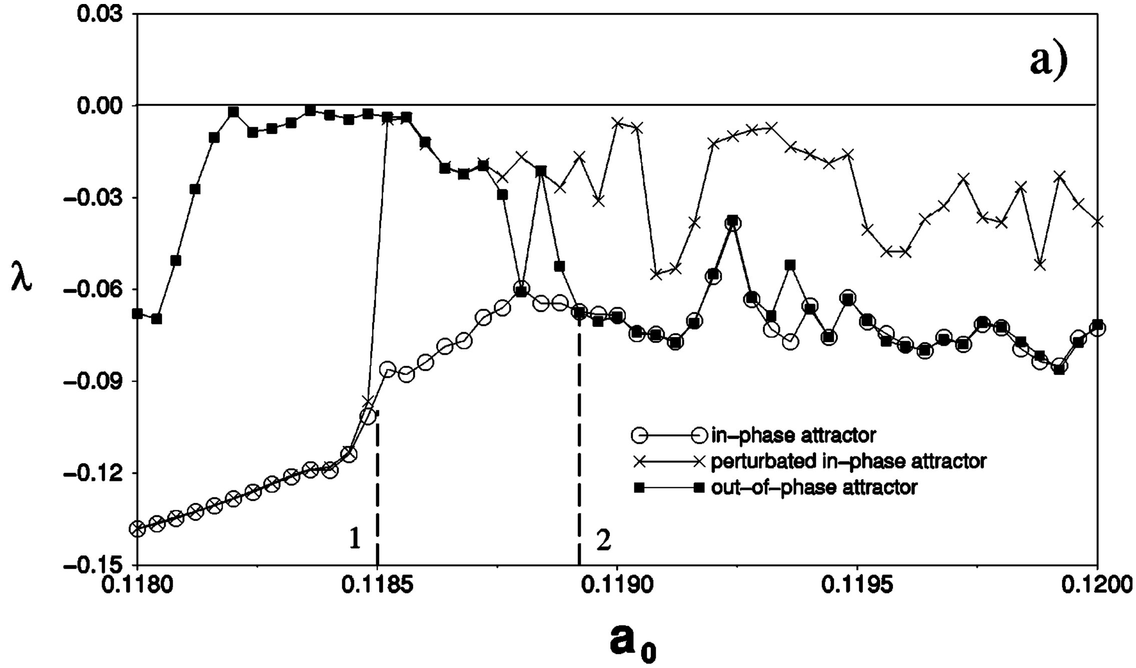


FIG. 1. xy projection of ''in-phase'' and ''out-of-phase'' strange nonchaotic attractors (SNA_0 and SNA_1, respectively) that coexist at $a_0=0.1186$, $p=1.0$.



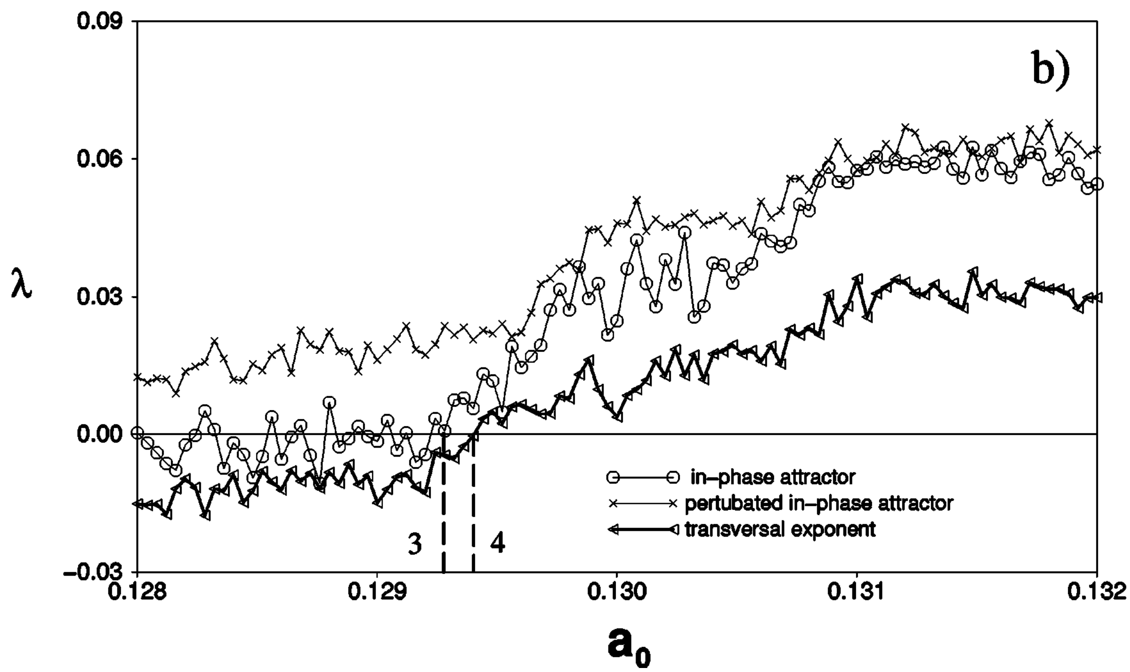


FIG. 2. Evolution of Lyapunov exponents vs the amplitude of the external forcing: (a) the largest Lyapunov exponent for two coexisting regimes and for the perturbated ''in-phase'' attractor; (b) the largest and transverse Lyapunov exponents for the ''in-phase'' regime. Perturbations are introduced as $p=1.0001$.

both attractors is diagnosed via criteria suggested by Pikovsky and Feudel [14]. The largest nonzero Lyapunov exponent for these attractors remains negative. Observed SNAs coexist within a certain range of the control parameter $a_0 \in [0.1185;0.1189]$ (Fig. 1).

Bifurcational transitions described above are easily distinguished by the calculation of the Lyapunov exponents as a function of the control parameter (Fig. 2). Stability of the ''in-phase'' regime is diagnosed with the help of the transversal Lyapunov exponent [10,11]:

$$\lambda_{tr}=\lim_{n\to\infty}\frac{1}{n}\ln\left|\prod_{i=0}^{n-1} f_u(u_i,v_i)\right|_{u_0=0}, \tag{2}$$

where $u=y-x$, $v=y+x$ are normalized variables, $f_u=\partial f/\partial u$. For the system (1), $f(u,v)=(2\gamma-1)uv$. To check the robustness of the ''in-phase'' regime, a small nonidentity of the partial systems is introduced. The ''in-phase'' regime of the SNA is shown to be structurally unstable to transverse perturbations. At the weak forcing mismatch $p=1.0001$, af-

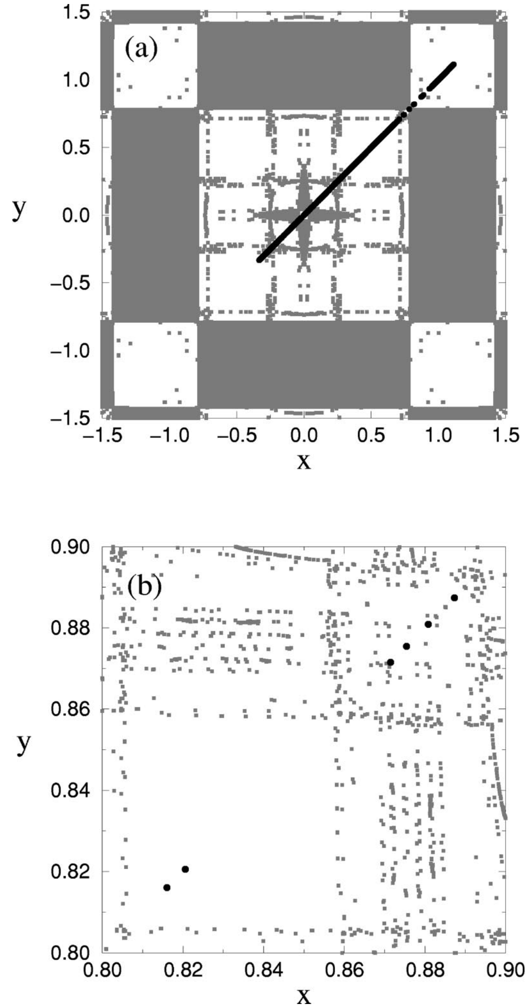


FIG. 3. (a) The stroboscopic section ($z = 0$) of attractors and their basins at $a_0 = 0.1186$. White color marks the basin of the ''in-phase'' state, while gray color indicates the basin of attraction of the ''out-of-phase'' attractor. Black dots belong to the ''in-phase'' regime.

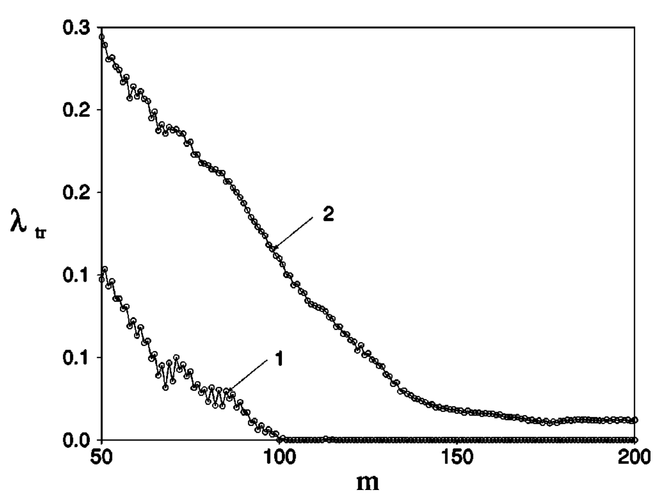


FIG. 4. The local transversal exponents vs the time of calculation for the quasiperiodic ''in-phase'' regime at $a_0 = 0.1184$ (curve 1) and for the strange nonchaotic set at $a_0 = 0.1186$ (curve 2).

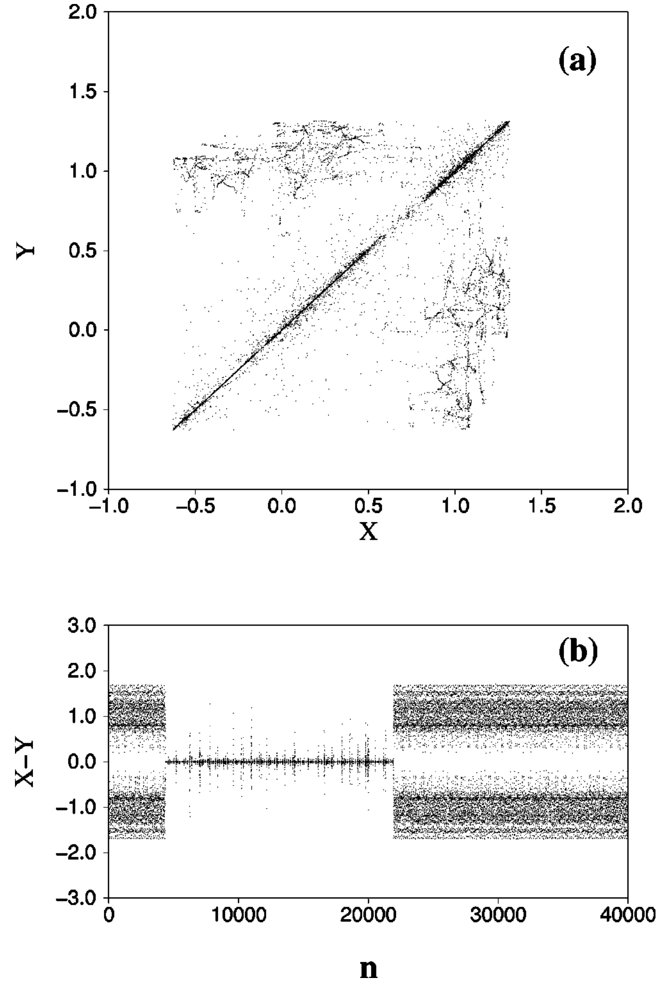


FIG. 5. On-off intermittency. (a) The phase portrait and (b) time series ($a_0 = 0.1190$, $p = 1.0001$).

ter long transient time phase trajectories leave the invariant manifold to an ''out-of-phase'' regime [point 1 in Fig. 2(a)]. Figure 3 represents the stroboscopic section of the basins of two coexisting strange nonchaotic attractors SNA_0 and SNA_1. It is clearly seen [Fig. 3(b)] that in a small vicinity of ''in-phase'' strange nonchaotic trajectories (because of finite time of calculation only a few points that belong to this attractor are indicated in Fig. 3(b) but it is known that the phase trajectories of a strange nonchaotic attractor are packed fully with nonuniform density [12]), there are a set of initial conditions starting from which trajectories will leave the neighborhood of the invariant manifold, approach an ''out-of-phase'' regime, and finally be attracted to it. The basin of the ''in-phase'' solution (white color in Fig. 3) is riddled by holes that are related to the basin of another attractor (gray color in Fig. 3). Therefore, the ''in-phase'' SNA appears to be a Milnor attractor [10,11,21].

Trajectories lying in the invariant subspace lose their transverse stability step by step. This process is characterized by the local transversal Lyapunov exponent, i.e., the transversal exponent that is calculated over a limit time interval (Fig. 4). One can see that in the case of the SNA (curve 2), trajectories suffer the local transversal instability during an essential larger number of iterations.

At $a_0 \approx 0.1189$, the ''out-of-phase'' attractor loses its stability [point 2 in Fig. 2(a)]. After a long transient period, a phase trajectory switches onto an ''in-phase'' regime, being a single attractive set in the phase space of the system. A weak forcing mismatch leads to on-off intermittency (Fig. 5) between two sets (the largest Lyapunov exponent oscillates around zero level) up to $a_0 \approx 0.130$ when an ''in-phase'' chaotic attractor appears [point 3 in Fig. 2(b)]. When the control parameter is slightly changed, the transverse Lyapunov exponent passes through zero from the negative side [point 4 in Fig. 2(b)]. This bifurcation is referred to as blowout bifurcation [8,19]. It is accompanied by the intermittency even without any perturbations and leads to the formation of a merged chaotic attractor including trajectories from ''in-phase'' and ''out-of-phase'' sets.

In summary, we have studied peculiarities of the interaction of quasiperiodically driven systems with the SNA. When a control parameter is varied, the system undergoes several transitions where the loss of complete synchronization for a strange nonchaotic regime is associated with the transverse destabilization, the appearance of a riddledlike structure in the basin of the attraction, and on-off intermittency. The bifurcation mechanism of the formation of such a structure for the SNA seems to be different from the known mechanism for a fully synchronized chaotic attractor, which we leave for further investigation.

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